Graph Theory: Week 6

Colourings

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Assignment 1

- Due in two weeks, October 9.
- Hand in at the lecture, or beforehand to my office at CIT, second floor.
- Each assignment makes up 20% of the final mark.

Recap: matchings

- Matchings
- The marriage theorem—condition for perfect matchings in a bipartite graph

Colourings

- A *colouring* of a graph $\mathcal{C}$ is an assignment of k colours to the edges of the graph.
- In a *proper* colouring, no two adjacent edges are the same colour.
- If G can be coloured with k colours, then we say it is k*-edge-colourable*.
- If k is the minimum number of colours for which this is possible, the graph is k*-edge-chromatic*.
- In this case, k is the *edge chromatic number*, $\chi(G)$.

A colouring as a set of matchings

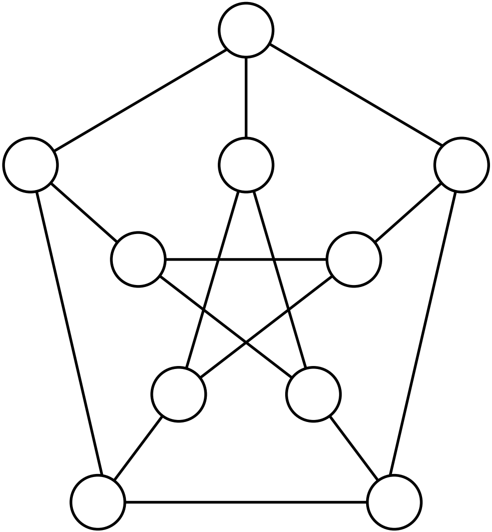
- Think of a colouring as a partitioning of the edges into subsets.
- Each subset contains no vertex no more than once.
- Therefore each subset is a matching.

What's the smallest edge colouring?

- $\chi(G)$ can't be less than the largest degree in the graph Δ.
- It also cannot be more than $\Delta + 1$ (Vizing's theorem).
- Therefore $\chi(G)$ is always equal to either Δ or $\Delta + 1$.
- A bipartite graph is always Δ-edge-chromatic.

Edge colouring in the Petersen graph

Show that this graph is 4-edge-chromatic:



Timetabling problem

- M teachers have to teach N classes.
- Every week, teacher T_i has to teach class C_j a total of $t_i j$ times.
- We need to work out what the most efficient timetable is.
- How can this problem be represented as a graph?
- How does edge colouring relate to this problem?

Algorithm challenge

Design an algorithm that finds a good colouring (using Δ colours) for a bipartite graph.