

Graph Theory: Assignment 1

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Due: Tues October 9th, 2007.

1. A simple connected graph G has ε edges and ν vertices.
 - (a) Let A and B be two vertices in the graph. If $\varepsilon = \nu + \alpha$, what is the maximum possible number of distinct paths between A and B ?
 - (b) In a tree, what is the maximum number of vertices with degree greater than one? Why?
 - (c) Use the previous result to show that the longest path in a tree must start and end with a vertex of degree one.
2. The following table shows the relative distances between five islands (one pair of islands cannot be travelled between directly due to visa problems).

	Lilliput	Brobdingnag	Laputa	Balnibarbi	Lindalino
Lilliput	-	4	5	5	9
Brobdingnag	4	-	2	8	2
Laputa	5	2	-	5	6
Balnibarbi	5	8	5	-	∞
Lindalino	9	2	6	∞	-

- (a) Use Dijkstra's algorithm to find the shortest route between Lilliput and Balnibarbi.
- (b) A system of shipping routes is to be designed in order to connect each place with maximum efficiency. Find the minimum spanning tree which connects all five destinations.
- (c) Adapt Kruskal's minimum spanning tree algorithm in order to make sure that there is a direct link between Balnibarbi and Brobdingnag. How does your new algorithm work, and what is the resulting spanning tree?
- (d) There is a problem with pirates in the seas around these islands. We now need an efficient system of shipping lines so that any one route can be disrupted without leaving any island disconnected. How would you calculate the new most efficient graph, and what is your result?